Hybridized Discretisation Methods for Computational Fluid Dynamics

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ABSTRACT

Hybrid discretization methods have recently gained attention for their ability to provide robust and accurate numerical solutions for partial differential equations. These methods are effective in both low- and high-order settings, making them well-suited for complex engineering problems. This talk will present recent advancement in two such approaches: the **face-centered finite volume (FCFV)** method [6] and the **hybridizable discontinuous Galerkin (HDG)** method [1,4], focusing on their application to compressible and incompressible flow simulations.

For large-scale flow simulations, industry demands fast and robust techniques capable of delivering results overnight. **FCFV** offers a competitive solution by handling general unstructured meshes while achieving first-order convergence of the stress tensor without requiring flux reconstruction. This talk will highlight common mesh-induced errors in traditional cell-centered finite volume (CCFV) methods. Unlike CCFV, FCFV is **less sensitive to mesh quality**, even in the presence of **highly distorted or stretched cells** [6,10]. Additionally, in **compressible flow** problems, FCFV provides **non-oscillatory** approximations of sharp discontinuities without the need for shock-capturing or limiting techniques [8]. For **transient or steady laminar and turbulent viscous incompressible** flows, novel convective stabilizations inspired by Riemann solvers enhance FCFV's **robustness and accuracy**, even under severe cell distortions in **high Reynolds number** boundary layers [10]. Moreover, FCFV remains stable in the incompressible limit, eliminating the need for special velocity-pressure coupling treatments [2,8,9,10].

For **high-fidelity simulations**, high-order methods are preferred due to their **low diffusion and dispersion errors**, though they often lack robustness. To address this, the development of **positivity-preserving and shear-preserving approximate Riemann solvers** for HDG discretizations will be discussed. The use of **HLL and HLLEM solvers** significantly improves shock wave resolution compared to traditional Lax-Friedrichs and Roe fluxes, particularly in **supersonic flows**, where physically admissible solutions are achieved without user-defined corrections [7]. Additionally, the high-order HDG framework demonstrates **robustness in the incompressible limit** and performs well for **weakly compressible flows in fluid-structure interaction problems**. Lastly, the benefits of **HDG-NEFEM** with **degree adaptivity** will be extended to simulations involving **NURBS-defined immersed boundaries and interfaces** [11].

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