Cutting the Meshing Bottleneck: NURBS-CutHDG for High Geometric Fidelity and High-Order Accuracy

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ABSTRACT

In many engineering CFD workflows, the primary practical bottleneck lies not in the solver but in generating analysis-ready meshes. Producing high-quality, body-fitted meshes is labor-intensive, difficult to automate reliably, and repeatedly time-consuming across iterative design cycles. This challenge is especially severe for curved, high-order meshes needed to represent complex CAD geometries with sufficient fidelity in applications ranging from aerospace to civil engineering (e.g., [1]). These costs motivate discretizations that decouple geometric fidelity from mesh generation, minimizing or eliminating manual meshing effort while maintaining high-order accuracy and robustness.

Hybrid discretization methods have attracted attention because they combine high-order accuracy with favorable linear-algebra properties. In particular, the Hybridizable Discontinuous Galerkin (HDG) method [2,6] supports degree-adaptive, high-order discretizations across a broad range of flow regimes (compressible or incompressible, viscous or inviscid), and reduces the globally coupled unknowns to the mesh skeleton via static condensation. When combined with the NURBS-enhanced finite element method (NEFEM) [7], HDG can represent CAD boundaries exactly, eliminating errors associated with piecewise-polynomial boundary approximations and, crucially, decoupling mesh size from boundary-feature resolution.

This talk presents the NURBS-CutHDG: an unfitted, degree-adaptive HDG formulation in which NURBS-defined boundaries and interfaces are embedded in a fixed background mesh (Cartesian or general). Accurate integration in cut elements is handled through NEFEM-based quadrature, and Dirichlet and Neumann boundary conditions are imposed consistently at any polynomial order without introducing additional degrees of freedom along non-conforming boundaries or interfaces. The resulting condensed global problem on faces/edges improves computational efficiency, supports scalable implicit solvers, and cleanly separates local element physics from global coupling.

The proposed method combines non-conforming grids, exact CAD-based geometry, and high-order accuracy to deliver robust predictions even on coarse meshes and in the presence of severely cut elements. Originally developed for two-fluid Stokes flows [8], the formulation extends naturally to incompressible (Navier–Stokes) and compressible (Euler) flows, while maintaining optimal convergence properties under challenging cut configurations. Numerical results confirm the method’s accuracy, stability, and effectiveness in capturing high-fidelity flow features.

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